1 Functions

Grad: $\nabla f = \frac{\partial f}{\partial x}\hat{e}_x + \frac{\partial f}{\partial y}\hat{e}_y + \frac{\partial f}{\partial z}\hat{e}_z$ Div: $\nabla \cdot f = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$

Laplace: $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Log-mean: $f_{\text{lm}}(x) = [f(x_2) - f(x_1)]/[\ln(f(x_2)/f(x_1))]$

2 Unsteady State Heat Transfer

$\rho \hat{C}_P \frac{DT}{Dt} = k\nabla^2 T - T\left(\frac{\partial P}{\partial T}\right)_\rho (\nabla \cdot \vec{v}) + \mu \Phi_{\text{v}}$ $\quad$ $\text{Bi} \equiv \frac{hV}{kA}$

Semi-inf. slab: $\frac{\partial T}{\partial t} = \alpha \nabla^2 T \quad \alpha = \frac{k}{\rho \hat{C}_P} \quad \frac{T-T_0}{T_1-T_0} = f\left(\frac{x}{\sqrt{2\alpha t}}, \frac{h\sqrt{\alpha t}}{k}\right)$

Wall: $\Phi_Q\big|_{x=0} = -k\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{k}{\sqrt{\pi \alpha t}}(T_1 - T_0) \quad \hat{Q} = 2(T_1 - T_0)\frac{k}{\sqrt{\pi\alpha}}\sqrt{t}$

3 Steady State Diffusion

Fick's 1st Law: $\frac{\boldsymbol{n}_{A,z}}{A} = cD_{AB}\frac{x_{A0}}{z}$

$\boldsymbol{n}'_{A,z} = -cD_{AB}\frac{\mathrm{d}x_A}{\mathrm{d}z} = -D_{AB}\frac{\mathrm{d}c_A}{\mathrm{d}z}$

$\boldsymbol{\Phi}_{n,A,z} = \underbrace{\boldsymbol{n}'_{A,z}}_{\text{diffusion}} + \underbrace{x_A(\boldsymbol{\Phi}_{n,A,z} + \boldsymbol{\Phi}_{n,B,z})}_{\text{convection}} = \boldsymbol{n}'_{A,z} + \frac{c_A}{c}(\boldsymbol{\Phi}_{n,A,z} + \boldsymbol{\Phi}_{n,B,z})$

$\boldsymbol{\Phi}_{n,A} = \frac{\dot{n}_A}{A}$

Ideal gas: $c_A = cx_A = \left[\frac{p_A}{RT} = \frac{n_A}{V}\right]$

Cyl. shell balance: $c = c_{\text{I}} + \frac{c_{\text{O}} - c_{\text{I}}}{\ln(R_{\text{O}}/R_{\text{I}})} \ln\frac{r}{R_{\text{I}}}$

3.1 Molecular Diffusion in Gases

EMCD: $\boldsymbol{\Phi}_{n,A,z} = \frac{D_{AB}(p_{A1} - p_{A2})}{RT(z_2 - z_1)}$

$\{\boldsymbol{\Phi}_{n,B} = 0\} : \boldsymbol{\Phi}_{n,A,z} = \frac{D_{AB}P(p_{A1} - p_{A2})}{RT(z_2 - z_1)p_{\text{lm},B}} \qquad p_{B,i} = P - p_{A,i}$

$D_{AA'} = \frac{1}{3}\langle\vec{v}\rangle\lambda = \frac{2\sqrt{\pi m_{\text{a},A} k_b T}}{3\pi^2 \varnothing_A^2 \rho} \qquad D_{AB} = \frac{\frac{2}{3}\sqrt{\frac{k_{\text{B}}T}{\pi}}\sqrt{0.5(\check{m}_A^{-1} + \check{m}_A^{-1})}}{\pi[0.5(\varnothing_A + \varnothing_B)]^2 n_A}$

Chapman-Enskog: $D_{AB} = \frac{1.8583\times10^{-7} T^{\frac{3}{2}}\sqrt{\check{m}_A^{-1} + \check{m}_B^{-1}}}{P\sigma_{AB}^2 \Omega_{D,AB}}$

$\sigma_{AB} = \frac{1}{2}(\sigma_A + \sigma_B)$

Fuller: $D_{AB} = \frac{1.0\times10^{-7} T^{1.75}\sqrt{\check{m}_A^{-1} + \check{m}_B^{-1}}}{P[(\sum v_A)^{\frac{1}{3}} + (\sum v_B)^{\frac{1}{3}}]^2}$

3.2 Molecular Diffusion in Liquids

EMCD: $\boldsymbol{\Phi}_{n,A,z} = \frac{\langle c\rangle D_{AB}(x_{A1} - x_{A2})}{z_2 - z_1}$

$\langle c\rangle = \frac{1}{N}\sum_{i=1}^{N}\frac{\rho_{zi}}{\langle\check{m}\rangle_{zi}} \qquad \langle\check{m}\rangle_{zi} = \left(\sum_j \frac{w_{j,zi}}{\check{m}_{j,zi}}\right)^{-1}$

$\{\boldsymbol{\Phi}_{n,B,z} = 0\} : \boldsymbol{\Phi}_{n,A,z} = \frac{\langle c\rangle D_{AB}(x_{A1} - x_{A2})}{(z_1 - z_2)x_{\text{lm},B}}$

Wilke-Chang: $D_{AB} = \frac{1.173\times10^{-16} T\sqrt{\phi \check{m}_B}}{\mu_B \check{V}_{\text{bp},A}^{0.6}}$

Diffusion cell:

$\boldsymbol{\Phi}_{n,A} = f_{\text{diff}} D_{AB}\frac{(c - c')}{\tilde{\tau}\delta} \qquad \ln\frac{c_0 - c_0'}{c - c'} = \frac{2f_{\text{diff}}A}{V\tilde{\tau}\delta} D_{AB} t$

$D_{AB} \propto \frac{T}{\mu_B} \to D_{AB}(T_2) = D_{AB}(T_1)\frac{T_2}{T_1}\frac{\mu_{B1}}{\mu_{B2}}$

3.3 Molecular Diffusion in Solids

$\boldsymbol{\Phi}_{n,A,z} = \frac{D_{AB}(c_{A1} - c_{A2})}{z_2 - z_1} = \frac{\underset{\cdot}{P}_{GS}(p_{A1} - p_{A2})}{(22.414\text{E–}3\ \text{m}^3\ \text{mol}^{-1})(z_2 - z_1)}$

$c_A = \frac{p_A \underset{\cdot}{s}}{22.414\text{E–}3\ \text{m}^3\ \text{mol}^{-1}}\ \left[\text{mol}_A\ \text{m}_{\text{solid}}^{-3}\right] \qquad \underset{\cdot}{P}_{GS} = D_{AB}\underset{\cdot}{s}$

Series: $\boldsymbol{\Phi}_{n,A,z} = \frac{p_{A1} - p_{A2}}{22.414\text{E–}3\ \text{m}^3\ \text{mol}^{-1}}\left(\sum_i \frac{\ell_i}{\underset{\cdot}{P}_{GS,i}}\right)^{-1}$

Porous: $\boldsymbol{\Phi}_{n,A} = \frac{\phi D_{AB}(c_{A1} - c_{A2})}{\tilde{\tau}(z_2 - z_1)}$

4 Unsteady State Diffusion

4.1 Flux Equations

A through non-diffusing B, or non-dilute A: $\boldsymbol{\Phi}_{n,A}(k') \to \boldsymbol{\Phi}_{n,A}(k)$

4.1.1 Gas EMCD or Dilute A

$\boldsymbol{\Phi}_{n,A} = k_c'(c_{A1} - c_{A2}) = k_G'(p_{A1} - p_{A2}) = k_y'(y_{A1} - y_{A2})$

$k_c' c = k_c'\frac{P}{RT} = k_c\frac{p_{\text{lm},B}}{RT} = k_G' P = k_G p_{\text{lm},B}$

$= k_y y_{\text{lm},B} = k_y' = k_c y_{\text{lm},B} c = k_G y_{\text{lm},B} P$

4.1.2 Liquid EMCD or Dilute A

$\boldsymbol{\Phi}_{n,A} = k_c'(c_{A1} - c_{A2}) = k_L'(c_{A1} - c_{A2}) = k_x'(x_{A1} - x_{A2})$

$k_c' c = k_L' c = k_L x_{\text{lm},B} c = k_L'\frac{\rho}{\check{m}} = k_x' = k_x x_{\text{lm},B}$

4.2 Diffusion, Convection, and Reaction

$\langle\vec{v}_m\rangle = \sum_i w_i \vec{v}_i \qquad w_i = \frac{\rho_i}{\rho}$	$\langle\vec{v}_n\rangle = \sum_i x_i \vec{v}_i \qquad x_i = \frac{c_i}{c}$
$\boldsymbol{m}_A = \rho_A(\vec{v}_A - \langle\vec{v}_m\rangle)$	$\boldsymbol{n}'_A = c_A(\vec{v}_A - \langle\vec{v}_n\rangle)$
$\boldsymbol{m}_A = -\rho D_{AB}\nabla w_A$	$\boldsymbol{n}'_A = -cD_{AB}\nabla x_A$
$\boldsymbol{\Phi}_{m,A} = \boldsymbol{m}_A + \rho_A\langle\vec{v}_m\rangle$	$\boldsymbol{\Phi}_{n,A} = \boldsymbol{n}'_A + c_A\langle\vec{v}_n\rangle$
$\frac{\partial \rho_A}{\partial t} + \nabla\cdot\boldsymbol{\Phi}_{m,A} = \check{R}_A$	$\frac{\partial c_A}{\partial t} + \nabla\cdot\boldsymbol{\Phi}_{n,A} = \check{r}_A$

$\underbrace{\frac{\partial c_A}{\partial t}}_{\text{accum.}} + \underbrace{\nabla\cdot c_A\langle\vec{v}_n\rangle}_{\text{convection}} - \underbrace{\nabla\cdot cD_{AB}\nabla x_A}_{\text{diffusion}} = \underbrace{\check{r}_A}_{\text{reaction}}$

$\{\Delta c, \Delta D_{AB} = 0\} : \frac{\partial c_A}{\partial t} + c_A(\nabla\cdot\langle\vec{v}_n\rangle) + \langle\vec{v}_n\rangle\cdot\nabla c_A - D_{AB}\nabla^2 c_A = \check{r}_A$

$\{\Delta\rho, \Delta D_{AB} = 0\} : \frac{\partial c_A}{\partial t} + \langle\vec{v}_n\rangle\cdot\nabla c_A - D_{AB}\nabla^2 c_A = \check{r}_A$

EMCD: $\frac{\partial c_A}{\partial t} = D_{AB}\nabla^2 c_A$

4.3 Cases

Catalytic converter: $r_A = \frac{-D_{AB} c_A \check{A}}{\delta} \qquad \check{A} \equiv \frac{A_{\text{wall}}}{\text{unit volume}}$

4.3.1 At Catalyst Surface with Gas Reaction $A \to 2B$

Instantaneous: $\boldsymbol{\Phi}_{n,A} = \frac{cD_{AB}}{\delta}\ln\frac{1 + x_{A1}}{1 + x_{A2}}$ $\qquad$ Slow: $x_{A2} \to \frac{\boldsymbol{\Phi}_{n,A}}{k_1' c}$

4.3.2 Atmospheric Dispersion

$\sigma_y^2 = \frac{2D_y x}{\vec{v}_x} \quad \sigma_z^2 = \frac{2D_z x}{\vec{v}_x} \quad \rho_A(x,y,z) = \frac{\dot{m}_A}{\pi\vec{v}_x\sigma_y\sigma_z} e^{\left[\frac{1}{2}\left(\frac{y^2}{\sigma_y^2} + \frac{(z-\hbar)^2}{\sigma_z^2}\right)\right]}$

$\rho_A(x,y,0) = \frac{\dot{m}_A}{\pi\vec{v}_x\sigma_y\sigma_z} e^{\frac{-\hbar^2}{2\sigma_z^2}} e^{\frac{-y^2}{2\sigma_y^2}} \qquad \rho_A(x,0,0) = \frac{\dot{m}_A}{\pi\vec{v}_x\sigma_y\sigma_z} e^{\frac{-\hbar^2}{2\sigma_z^2}}$

4.3.3 Diffusion-Adsorption in Capping Sediment Layer

$\boldsymbol{\Phi}_{n,A} = \frac{D_{\text{eff}}\rho_A^*}{a}\left[1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-\frac{D_{\text{eff}}t}{f_{\text{r}}}\left(\frac{n\pi}{a}\right)^2}\right]$

$a \equiv$ Effective cap thickness
$f_{\text{r}} \equiv$ Retardation factor
$\rho^* \equiv$ Solubility limit

4.4 Coefficients for Various Geometries

$\text{Sc} \equiv \frac{\mu}{\rho D_{AB}} \quad \text{Re} \equiv \frac{\rho\vec{v}\ell'}{\mu} \quad \text{Sh} \equiv \frac{k_c'\ell'}{D_{AB}} \quad \text{St} \equiv \frac{k_c'}{\vec{v}} \quad \ell' \in \{\varnothing, \delta, \ell\}$

$J_D = \text{St}\,\text{Sc}^{\frac{2}{3}} = \frac{k_c'}{\vec{v}}\text{Sc}^{\frac{2}{3}} = \frac{\text{Sh}}{\text{Re}\,\text{Sc}^{\frac{1}{3}}}$

Flow $\parallel$ flat plate: $J_D = a\text{Re}^b\text{Sc}^c$

Gas, $\{\text{Re} < 15000\} : a = 0.664,\ b = 0.5,\ c = 1/3$

$\{15000 < \text{Re} < 300000\} : a = 0.036,\ b = -0.2,\ c = 0$

Liq., $\{600 < \text{Re} < 50000\} : a = 0.99,\ b = -0.5,\ c = 0$

5 Evaporation

$\dot{Q} = UA\Delta T = S\lambda_S \qquad \lambda_S = \hat{H}_S - \hat{h}_S \qquad \text{Stm. economy} \equiv \frac{m_{\text{vap}}}{m_{\text{stm}}}$

Total: $F = L + V$ $\qquad$ Solute: $Fx_F = Lx_L + Vy_V$

$Fh_F + SH_S = Lh_L + VH_V + Sh_S \qquad Fh_F + S\lambda_S = Lh_L + VH_V$

Single: $h_F = \hat{C}_{P,F}(T_F - T_{\text{bp}}) \qquad h_L = \hat{C}_{P,L}(T_L - T_{\text{bp}})$

Mult.: $\sum_{i=1}^{n}\Delta T_i = T_S - T_n \qquad \Delta T_j = \left[\sum_{i=1}^{n}\Delta T_i\right]\frac{U_j^{-1}}{\sum_{i=1}^{n}(U_i^{-1})}$

$U_{\text{I}} = \left(\frac{1}{h_{\text{I}}} + \frac{(R_{\text{O}} - R_{\text{I}})A_{\text{I}}}{k_A A_{\text{lm}}} + \frac{A_{\text{I}}}{A_{\text{O}}h_{\text{O}}}\right)^{-1}$

$U_{\text{O}} = \left(\frac{A_{\text{O}}}{A_{\text{I}}h_{\text{I}}} + \frac{(R_{\text{O}} - R_{\text{I}})A_{\text{O}}}{k_A A_{\text{lm}}} + \frac{1}{h_{\text{O}}}\right)^{-1}$

$A_{\text{lm}} = \frac{A_{\text{O}} - A_{\text{I}}}{\ln(A_{\text{O}}/A_{\text{I}})} \qquad A_i = 2\pi R_i \ell$

6 Drying

6.1 Humidity

$\underset{\cdot}{H} = \frac{\check{m}_{(\text{l})}}{\check{m}_{\text{da}}}\frac{p_{(\text{l})}}{P - p_{(\text{l})}} \qquad \underset{\cdot}{H}_{\text{S}} = \frac{\check{m}_{(\text{l})}}{\check{m}_{\text{da}}}\frac{p_{(\text{l})}^\star}{P - p_{(\text{l})}^\star} \qquad \underset{\cdot}{H}_{\text{P}} = \frac{100\underset{\cdot}{H}}{\underset{\cdot}{H}_{\text{S}}} \qquad \underset{\cdot}{H}_{\text{R}} = \frac{100p_{(\text{l})}}{p_{(\text{l})}^\star}$

$V_H = \frac{22.41}{273.15}T(\check{m}_{\text{da}}^{-1} - \check{m}_{(\text{l})}^{-1}\underset{\cdot}{H}) \qquad \rho_H = \frac{1\text{ kg da} + \underset{\cdot}{H}}{V_H}$

$C_H = C_{P,\text{da}} + C_{P,(\text{l})}\underset{\cdot}{H} \qquad \hat{H}_y = C_H(T - T_0) + \underset{\cdot}{H}\lambda_0$

Water/air: $\hat{H}_y = (1.005 + 1.88\underset{\cdot}{H})(T - T_0) + \underset{\cdot}{H}\lambda_0 \quad \lambda_0(0\,°\text{C}) = 2501.4\ \frac{\text{J}}{\text{g}}$

6.2 Batch Drying

$L_S = A\delta\rho_S \qquad \vec{G} = \vec{v}\rho_H \qquad \vec{G}[=]\frac{\text{kg}}{\text{h m}^2} \qquad \vec{v}[=]\frac{\text{m}}{\text{s}}$

Gas$\|$sfc, $\{318 \le T \le 423\}$,

$\{2450 \le \vec{G} \le 29300\} \vee \{0.61 \le \vec{v} \le 7.6\} : h = 0.0204\vec{G}^{0.8}$

Gas$\perp$sfc, $\{3900 \le \vec{G} \le 19500\} \vee \{0.9 \le \vec{v} \le 4.6\} : h = 1.17\vec{G}^{0.37}$

$\frac{\underset{.}{H}-\underset{.}{H}_S}{T-T_{wb}} = -\frac{r_{psychro}}{\lambda_{(l)}} \qquad r_{psychro} \equiv \frac{h}{\check{m}_{da}k_y}$

$\dot{D} = \frac{L_S}{A}\frac{d\chi}{dt} \qquad \dot{D}_c = \frac{\dot{Q}}{A\lambda_{(l)}} = k_y\check{m}_{da}(\underset{.}{H}_S - \underset{.}{H}) = \frac{h(T-T_{wb})}{\lambda_{(l)}}$

$\chi = \frac{m_{wet}-m_S}{m_S} \qquad t_{drying} = \frac{L_S(\chi_1-\chi_2)}{A\dot{D}_c} = \frac{L_S(\chi_1-\chi_2)}{Ak_y\check{m}_{da}(\underset{.}{H}_S-\underset{.}{H})}$

Falling rate in capillary: $t(\chi_c,\chi) = \frac{\delta\rho_S\lambda_{(l)}\chi_c}{h(T_{wb}-T)}\ln\frac{\chi_c}{\chi}$

6.3 Continuous Drying

Moisture: $G\underset{.}{H}_2 + \dot{L}_S\chi_1 = G\underset{.}{H}_1 + \dot{L}_S\chi_2 \qquad Gd\underset{.}{H} = \dot{L}_S d\chi$

Heat: $G\hat{H}_{G2} + \dot{L}_S\hat{H}_{S1} = G\hat{H}_{G1} + \dot{L}_S\hat{H}_{S2} \underset{gain}{\overset{lose}{\pm}} \dot{Q}$

$\hat{H}_G = C_H(T_G - T_0) + \underset{.}{H}\lambda_0$

$\hat{H}_S = \hat{C}_{P,S}(T_S - T_0) + \chi\hat{C}_{P,L}(T_S - T_0)$

Countercurrent: $\dot{L}_S(\chi_c - \chi_2) = G(\underset{.}{H}_c - \underset{.}{H}_2)$

$t_{const} = \frac{G}{\dot{L}_S}\frac{\dot{L}_S}{A}(k_y\check{m}_{da})^{-1}\ln\left(\frac{\underset{.}{H}_S-\underset{.}{H}_c}{\underset{.}{H}_S-\underset{.}{H}_1}\right)$

$t_{falling} = \frac{G}{\dot{L}_S}\frac{\dot{L}_S}{A}\frac{\chi_c}{k_y\check{m}_{da}}\left[\frac{(\underset{.}{H}_S-\underset{.}{H}_2)G}{\dot{L}_S}+\chi_2\right]^{-1}\ln\left[\frac{\chi_c(\underset{.}{H}_S-\underset{.}{H}_2)}{\chi_2(\underset{.}{H}_S-\underset{.}{H}_c)}\right]$

7 Countercurrent Multiple-Contact Gas-Liquid Separation

$p_A = \mathcal{H}x_A \qquad y_A = \mathcal{H}'x_A \qquad \mathcal{H}' \equiv \mathcal{H}/P$

Total: $L_0V_{N+1} = L_N + V_1 \quad A$: $L_0x_0 + L_{N+1}y_{N+1} = L_Nx_NV_1y_1$

n: $L_0 + V_{n+1} = L_n + V_1 \quad A$: $L_0x_0 + V_{n+1}y_{n+1} = L_nx_nV_1y_1$

Op. line: $y_{n+1} = \frac{L_n}{V_{n+1}}x_n + \frac{V_1y_1-L_0x_0}{V_{n+1}}$ Eq. line: $y_n = \mathcal{H}'x_n$

Stripping: $N = \frac{\ln\left[\frac{x_0-\frac{y_{N+1}}{m}}{x_N-\frac{y_{N+1}}{m}}(1-f_{abs})+f_{abs}\right]}{\ln\left(f_{abs}^{-1}\right)}$

$f_{abs} \equiv \frac{L}{mV} \qquad f_{a,N} \equiv \frac{L_N}{mV_{N+1}} \qquad m \equiv$ slope of eq. line

Absorption: $N = \frac{\ln\left[\frac{y_{N+1}-mx_0}{y_1-mx_0}(1-f_{str})+f_{str}\right]}{\ln\left(f_{str}^{-1}\right)} \qquad f_{str} \equiv \frac{mV}{L} = f_{abs}^{-1}$

8 Continuous Humidification Process

$\Phi_L\hat{C}_{P,L}dT_L = \frac{\dot{Q}_{lat}}{A} + \frac{\dot{Q}_{sens}}{A}$

$\frac{\dot{Q}_{lat}}{A} = \check{m}_{da}k_G\check{A}P\lambda_0(\underset{.}{H}_{int} - \underset{.}{H}_G)d\hbar \qquad \frac{\dot{Q}_{sens}}{A} = h_G\check{A}(T_{int} - T_G)d\hbar$

$\Phi_L\hat{C}_{P,L}dT_L = \Phi_G d\hat{H}_y = (h_L\check{A})d\hbar(T_L - T_{int})$

Heat: $\Phi_G(\hat{H}_{y2} - \hat{H}_{y1}) = \Phi_L\hat{C}_{P,L}(T_{L2} - T_{L1})$

$\Phi_G \approx \frac{3}{2}\Phi_{G,min} \qquad \Phi_{G,min} = \frac{\Phi_L\hat{C}_{P,L}}{\Delta\hat{H}_y/\Delta T_L} \qquad \frac{\hat{H}_{y,int}-\hat{H}_y}{T_{int}-T_L} = \frac{-h_L\check{A}}{\check{m}_{da}k_G\check{A}P}$

Film: $\hbar = \hbar_G\int_{\hat{H}_{y1}}^{\hat{H}_{y2}}\frac{d\hat{H}_y}{\hat{H}_{y,int}-\hat{H}_y} \qquad \hbar_G = \frac{\Phi_G}{\check{m}_{da}k_G\check{A}P}$

Overall : $\bar{\hbar} = \bar{\hbar}_G\int_{\hat{H}_{y1}}^{\hat{H}_{y2}}\frac{d\hat{H}_y}{\hat{H}_y^*-\hat{H}_y} \qquad \bar{\hbar}_G = \frac{\Phi_G}{\check{m}_{da}\bar{k}_G\check{A}P}$

$H_y^* \equiv H_y$ on eq. line at temp. T from op. line

9 Adsorption Process: A adsorbs B

$q \equiv \frac{m_{adsorbate}}{m_{adsorbent}} \qquad C \equiv \frac{m_{adsorbate}}{V_{fluid}} \qquad K, n, q_0 \equiv$ emp. constants

Isotherms:

Linear: $q = KC_B \qquad K[=]\frac{m^3}{kg_{adsorbent}}$

Freundlich: $q = KC_B^n \qquad n = \frac{\Delta\log q}{\Delta\log c} \qquad K[=]$varies

Langmuir: $q = \frac{q_0C_B}{K+C_B} \qquad q_0[=]\frac{kg_{adsorbate}}{kg_{solid}} \qquad K[=]\frac{kg}{m^3}$

Batch: $q_Fm_A + C_{B,F}V_F = q_fm_A + C_{B,f}V_F$

9.1 Fixed Bed Adsorption Columns

$t_T = \int_0^\infty(1 - \frac{C}{C_0})dt \qquad t_{usable} = \int_0^{t_{breakpoint}}(1 - \frac{C}{C_0})dt$

$f_{capacity} = \frac{t_{usable}}{t_T} \qquad \hbar_{breakpoint} = \frac{t_{usable}}{t_T}\hbar_T$

$\phi\frac{\partial C}{\partial t} + (1-\phi)\rho_p\frac{\partial q(C)}{\partial t} = -\vec{v}_s\frac{\partial C}{\partial z} + D_a\frac{\partial^2 C}{\partial z^2}$

$-\frac{dz}{dt} = -\frac{\partial C/\partial t}{\partial C/\partial z} = \frac{\vec{v}_s}{\phi+(1-\phi)\rho_pq(C)}$

10 Membrane Separation

10.1 Liquid Permeation Membrane

$K' \equiv \frac{c_{m:f}}{c_{f:m}}$ m: membrane, f: film

$\mathbf{\Phi}_{n,A} = k_{c1}(c_1 - c_{1,f:m}) = \frac{D_{AB}}{\delta}(c_{1,m:f} - c_{2,m:f}) = k_{c2}(c_{2,f:m} - c_2)$

$\mathbf{\Phi}_{n,A} = \frac{c_1-c_2}{k_{c1}^{-1}+\underset{.}{P}_L{}^{-1}+k_{c2}^{-1}} \qquad \underset{.}{P}_L = \frac{D_{AB}K'}{\delta_{mem}}$

10.2 Gas Permeation Membrane

$\mathbf{\Phi}_{n,A} = \frac{p_{A1}-p_{A2}}{\frac{RT}{k_{c1}}+\frac{\delta_{mem}}{\underset{.}{P}_{G,A}}+\frac{RT}{k_{c2}}} \qquad \underset{.}{P}_G = \frac{D_{AB}\underset{.}{s}}{22.414E{-}3\ m^3\ mol^{-1}}$

10.2.1 Complete Mixing Model

$\dot{V}_0 = \dot{V}_R + \dot{V}_P \qquad \dot{V}_0y_0 = \dot{V}_Ry_R + \dot{V}_Py_P \qquad \theta \equiv \frac{\dot{V}_P}{\dot{V}_0}$

$\frac{\dot{V}_{P,A}}{A_{mem}} = \frac{\dot{V}_Py_P}{A_{mem}} = \frac{\underset{.}{P}_{GS,A}}{\delta_{mem}}(P_Hy_R - P_Ly_P)$

$\frac{\dot{V}_{P,B}}{A_{mem}} = \frac{\dot{V}_P(1-y_P)}{A_{mem}} = \frac{\underset{.}{P}_{GS,B}}{\delta_{mem}}[P_H(1-y_R) - P_L(1-y_P)]$

$\alpha^\circ \equiv \frac{\underset{.}{P}_{GS,A}}{\underset{.}{P}_{GS,B}} \qquad r \equiv \frac{P_L}{P_H} \qquad \frac{y_P}{1-y_P} = \frac{\alpha^\circ[y_R-ry_P]}{(1-y_R)-r(1-y_P)}$

$y_P = \frac{-b+\sqrt{b^2-4ac}}{2a}$

$y_0, y_R, \alpha^\circ, r$ known:

$a \equiv 1-\alpha^\circ \qquad b \equiv -1+\alpha^\circ+\frac{1}{r}+\frac{y_R}{r}(\alpha^\circ-1) \qquad c \equiv \frac{-\alpha^\circ y_R}{r}$

$y_0, \theta, \alpha^\circ, r$ known:

$a \equiv \theta + r - r\theta - \alpha^\circ\theta - \alpha^\circ r + \alpha^\circ r\theta$

$b \equiv 1 - \theta - y_0 - r + r\theta + \alpha^\circ\theta + \alpha^\circ r - \alpha^\circ r\theta + \alpha^\circ y_0 \qquad c \equiv -\alpha^\circ y_0$

$y_R = \frac{y_0-\theta y_P}{1-\theta} \qquad y_P = \frac{y_0-y_R(1-\theta)}{\theta} \qquad A_{mem} = \frac{\theta\dot{V}_0y_P}{\frac{\underset{.}{P}_{GS,A}}{\delta_{mem}}(P_Hy_R-P_Ly_P)}$

$\{\theta = 1\}: \ y_{R,min} = \frac{y_0[1+r(\alpha^\circ-1)(1-y_0)]}{\alpha^\circ(1-y_0)+y_0}$

10.3 Reverse Osmosis Membrane: A dissolves B

$\Pi = \frac{n_BRT}{V_A} \qquad \underset{.}{P}_{slv,A} = \frac{D_A\langle C_{mem}\rangle\check{V}_A}{RT}$

$\underset{.}{P}_{\delta,A} \equiv \frac{\underset{.}{P}_{slv,A}}{\delta_{mem}} \qquad D_{\delta,B} \equiv \frac{D_{mem,B}K'_B}{\delta_{mem}} \qquad K'_B \equiv \frac{C_{B,mem}}{C_B}$

$\mathbf{\Phi}_{m,A} = \underset{.}{P}_{\delta,A}(\Delta P - \Delta\Pi) \qquad \Delta : 1 \xrightarrow{membrane} 2$

$\mathbf{\Phi}_{m,B} = D_{\delta,B}(C_{B1} - C_{B2}) = \frac{\mathbf{\Phi}_{m,A}C_{B2}}{C_{A2}}$

$r_{reject} \equiv 1 - \frac{C_{B2}}{C_{B1}} = \frac{B(\Delta P-\Delta\Pi)}{1+B(\Delta P-\Delta\Pi)} \qquad B \equiv \frac{\underset{.}{P}_{slv,A}}{D_{mem,B}K'_BC_{A2}} = \frac{\underset{.}{P}_{\delta,A}}{D_{\delta,B}C_{A2}}$

11 Mechanical Separation

11.1 Constant Pressure Filtration

Carman-Kozeny: $\frac{\Delta P_{cake}}{\delta} = \frac{k_1\mu\vec{v}_s(1-\phi)^2\check{A}_p^2}{\phi^3} \qquad k_1 = 4.17 \qquad \check{A}_p \equiv \frac{A_{S,p}}{V_p}$

$\frac{dt}{dV} = K_PV + B \qquad K_P \equiv \frac{\mu\alpha c_S}{A^2(-\Delta P)}\left[\frac{s}{m^6}\right] \qquad B \equiv \frac{\mu R_f}{A(-\Delta P)}\left[\frac{s}{m^3}\right]$

$\alpha \equiv \frac{k_1(1-\phi)\check{A}_p^2}{\rho_p\phi^3} \qquad \frac{t}{V} = \frac{K_PV}{2} + B \qquad K_P \propto A^{-2} \qquad B \propto A^{-1}$

$t_{fltrn} = \frac{K_PV^2}{2} + BV$

Leaf: $\left(\frac{dV}{dt}\right)_{wash} = \frac{1}{K_PV_T+B}$ Plate-frame: $\frac{1}{4}$ leaf $\qquad t_{wash} = \frac{V_{wash}}{\dot{V}_{wash}}$

11.2 Centrifugation

$\vec{F} = m\vec{a}_r = mR\omega^2 \qquad \omega = \frac{\vec{v}_t}{r} \qquad \omega = \frac{2\pi\dot{N}_r}{60}$

$\{Re < 1\}: \ \vec{v}_\infty = \frac{\omega^2\varnothing_p^2 r(\rho_p-\rho)}{18\mu}$

$\dot{V}_c = \frac{\omega^2\varnothing_{p,c}^2(\rho_p-\rho)}{18\mu\ln[2R_2/(R_1+R_2)]}V \qquad V = \pi\hbar_{bowl}(R_2^2 - R_1^2) \qquad t_T = \frac{V}{\dot{V}_c}$

$\frac{\dot{V}_1}{\Sigma_1} = \frac{\dot{V}_2}{\Sigma_2}$ 1:small scale 2:process scale $\qquad \Sigma \equiv \frac{\omega^2[\pi\hbar_{bowl}(R_2^2-R_1^2)]}{2g\ln[2R_2/(R_1+R_2)]}$

References

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